S-DUALITY AND THE ENTROPY OF BLACK HOLES IN HETEROTIC STRING THEORY

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April 30, 1995

Abstract

Four dimensional heterotic string effective action is known to admit non-rotating electrically and magnetically charged black hole solutions. It is shown that the partition functions and entropies for both the cases are identical when these black hole solutions are related by S-duality transformations. The entropy is computed and is vanishing for each black hole in the extremal limit.

PACS NO. 04.70.Dy, 04.60.-m, 04.40.Nr

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Recently, Hawking and Ross [1] have implemented S-duality transformation to derive interesting results for the electrically and magnetically charged black holes. It is well known that the partition function for electrically charged black holes is analogous to that of the grand canonical ensemble with a chemical potential; whereas the partition function for the magnetically charged black holes has the interpretation of canonical partition function. Naturally, the two partition functions differ from one another. In order to investigate the implications of S-duality the charged black hole partition function is projected to one with definite charge and then it is found that the partition functions for the electrically and magnetically charged black holes are identical.

It is recognized that the consequences of S-duality are interesting and surprising [2]. In the recent past, considerable attention has been focussed to investigate implications of S-duality in sring theory [3] and supersymmetric field theories [4]. This transformation, in its simplest form, when applied to electrodynamics, interchanges the roles of electric and magnetic fields. However, in theories with larger number of field contents, the transformation rules for the fields are to be appropriately defined so that the equations of motion remain invariant under S-duality transformations. Note however, that the action is not necessarily invariant under these transformtions. If we toroidally compactify ten dimensional heterotic string effective action [5] to four dimensions, the reduced effective action is endowed with N=4 supersymmetry. One of the attractive features of the four dimensional theory is that the quantum corrections are restricted due to the existence of non-renormalisation theorems. It is expected that this theory respects strong-weak coupling duality symmetry and might provide deeper insight into the non-perturbative aspects of string theory. Furthermore, the four dimensional reduced action can be cast in a manifestly O(6,22) invariant form. Cosequently, one can generate new background field configurations by implementing suitable O(6,22) and S-duality

tranformations on a known solution of the effective action. String effective action admit black hole solutions[6]. Sen [7] has obtained interesting solutions such that they preserve N=4 supersymmetry, saturating Bogomol'nyi bound in the extremal limit.

The purpose of this letter is to investigate the consequences of S-duality transformations for the black hole solutions of the four dimensional heterotic string effective action and investigate the thermodynamic properties of these black holes. First, we compute the partition function of an electrically charged black hole which resembles a grand partition function with a chemical potential. Then we adopt the techniques due to Coleman Preskill and Wilczek [8] to project to a partition function with definite charges. The free energy, entropy and the thermodynamical potential are derived. The entropy is shown to be zero in the extremal limit. Our result is of importance due to the fact that these extremal black hole solutions saturate Bogomol'nyi bound and the partition functions are expected not to get quantum corrections due to N=4 supersymmetry. Our next result provide yet another proof of S-duality. We envisage electrically charged black hole solutions with zero axion field. Subsequently we implemented the S-duality tranformation such that the electrically charged solutions go over to the magnetic ones. Furthermore, the charged projected partition function for electrically charged black hole is identical to that of the magnetically charged black hole. While calculating the partition functions we use the Euclidean formulation throughout the paper.

The massless bosonic sector of four dimensional heterotic string effective action consist of dialton, graviton, axion (dual to the antisymmetric tensor field) and 28 abelian gauge fields denoted respectively by ϕ, $G_{\mu\nu}$, ψ and $A_{\mu}^{(a)}$, $a = 1 \cdots 28$. The effective action, written in Einstein metric, $g_{\mu\nu} = e^{-\phi} G_{\mu\nu}$ takes the following form,

$$S = \int d^4x \sqrt{-g} \Big\{ R + \frac{1}{2\lambda_2^2} \partial_\mu \bar{\lambda} \partial^\mu \lambda + \frac{1}{8} \text{tr}(\partial_\mu ML \partial^\mu ML)$$

3

$$- \lambda_2 F_{\mu\nu}^{(a)}(LML)_{ab}F^{(b)\mu\nu} + \lambda_1 F_{\mu\nu}^{(a)} L_{ab} \tilde{F}^{(b)\mu\nu} \Big\} + \quad \text{Boundary\ terms} \tag{1}$$

Where,

$$F_{\mu\nu}^{(a)} = \partial_\mu A_\nu^{(a)} - \partial_\nu A_\mu^{(a)}. \tag{2}$$

M and L are 28×28 symmetric matrices; whereas, M paramtrizes the coset $\frac{O(6,22)}{O(6)\otimes O(22)}$, L has 22 eigenvalues -1 and 6 eigenvalues $+1$ and has the following form

$$L = \begin{pmatrix} -I_{22} & 0 \\ 0 & I_6 \end{pmatrix}. \tag{3}$$

M satisfies the property, $MLM^T = L$. λ is a complex scalar field defined in terms of the axion and the dilaton $\lambda = \psi + ie^{-\phi}$. The equations of motion associated with (1) remain invariant under the S-duality transformations

$$\lambda \rightarrow \lambda' = \frac{a\lambda + b}{c\lambda + d}, \qquad ad - bc = 1$$
$$F_{\mu\nu}^{(a)} \rightarrow (c\lambda_1 + d)F_{\mu\nu}^{(a)} + c\lambda_2(ML)_{ab}\tilde{F}_{\mu\nu}^{(a)}. \tag{4}$$

where a,b,c and d are real. The backgrounds $g_{\mu\nu}$ and M remain invariant under the transformation.

The invariance property of the action under the non-compact global transformations has been utilised by Sen to generate charged rotating black hole solutions [7] starting from an uncharged rotating black hole solution with $\phi = 0$, $B_{\mu\nu} = 0$, $A_\mu^{(a)} = 0$, $M = I_{28}$. A special case corresponding to non-rotating charged black hole is given by the following background configurations,

$$ds^2 = -\Delta^{-1/2}(r^2 - 2mr)dt^2 + \Delta^{1/2}(r^2 - 2mr)^{-1}dr^2 + \Delta^{1/2}d\Omega_{II}^2 \tag{5}$$

where, $\Delta = r^4 + 2mr^3(\cosh\alpha\cosh\gamma - 1) + m^2r^2(\cosh\alpha - \cosh\gamma)^2$. The dilaton is given by,

4

$$e^\phi = r^2/\Delta^{1/2} \tag{6}$$

The gauge fields are,

$$
\begin{aligned}
A_t^{(a)} &= -\frac{n^{(a)}}{\sqrt{2}} \frac{mr}{\Delta} \sinh\alpha [r^2 \cosh\alpha + mr(\cosh\alpha - \cosh\gamma)] \qquad 1 \le a \le 22 \\
&= -\frac{p^{(a-22)}}{\sqrt{2}} \frac{mr}{\Delta} \sinh\gamma [r^2 \cosh\alpha + mr(\cosh\gamma - \cosh\alpha)] \qquad 23 \le a \le 28
\end{aligned} \tag{7}
$$

The M-matrix, satisfying the equations of motion is,

$$
M = I_{28} + \begin{pmatrix} Pnn^T & Qnp^T \\ Ppn^T & Ppp^T \end{pmatrix} \tag{8}
$$

where, $P = 2\frac{m^2 r^2}{\Delta} \sinh^2\alpha \sinh^2\gamma$ and $Q = 2\frac{mr}{\Delta} \sinh\alpha \sinh\gamma [r^2 + mr(\cosh\alpha \cosh\gamma - 1)]$.

The mass of this black hole is

$$
M = \frac{m}{2}(1 + \cosh\alpha \cosh\gamma) \tag{9}
$$

The 28 charges are given by

$$
\begin{aligned}
Q^{(a)} &= \frac{n^{(a)}}{\sqrt{2}} m \sinh\alpha \cosh\gamma \qquad 1 \le a \le 22 \\
&= \frac{p^{(a-22)}}{\sqrt{2}} m \sinh\gamma \cosh\alpha \qquad 23 \le a \le 28
\end{aligned} \tag{10}
$$

Note that the "boost" angles α, γ are the parameters [7,9] that appear in global non-compact transformation to obtain charged solution from the uncharged one.

The horizon of this black hole is at $r = 2m$, there is a curvature singularity at $r = 0$, the area of the event horizon is $A_H = 8\pi m^2(\cosh\alpha + \cosh\gamma)$ and the Hawking-temperature is given by $T_H = \frac{1}{4\pi m(\cosh\alpha + \cosh\gamma)}$.

There are two extremal limits of this black hole which preserve supersymmetry and consequently saturate the Bogomol'nyi bound for the mass:

(I) $m \to 0, \gamma \to \infty$, while keeping $m \cosh\gamma = m_0$ finite and α is finite but arbitrary. For this case $\Delta = r^2(r^2 + 2m_0 r \cosh\alpha + m_0^2)$, the mass is $M = \frac{m_0}{2}\cosh\alpha$ and the charges are

5

$$Q_L^{(a)} = \frac{n^{(a)}}{\sqrt{2}} m_0 \sinh \alpha \quad 1 \leq a \leq 22, \qquad Q_R^{(a)} = \frac{p^{(a-22)}}{\sqrt{2}} m_0 \cosh \alpha \quad 23 \leq a \leq 28 \qquad (11)$$

The horizon of this black hole is at $r = 0$, hence $A_H = 0$ and temperature , $T_H = \frac{1}{8\pi M} \cosh \alpha$.

The expression for the gauge fields become,

$$A_{tL}^{(a)} = -\frac{n^{(a)}}{\sqrt{2}} \frac{mr^3}{\Delta} \sinh \alpha \quad 1 \leq a \leq 22, \quad A_{tR}^{(a)} = -\frac{p^{(a-22)}}{\sqrt{2}} \frac{mr^3}{\Delta} \cosh \alpha \quad 23 \leq a \leq 28 \quad (12)$$

The Bogomol'nyi bound is saturated and $M^2 = \frac{1}{2} \vec{Q}_R^2$, where R stands for the right hand sector and the index a runs from 23 to 28. Also $Q_L^{(a)} = \sqrt{2} M n^{(a)} \tanh \alpha$.

(II) The other black hole corresponds to limits: $m \to 0$, $\alpha = \gamma \to \infty$ such that $m \cosh^2 \alpha = m_0$ is finite. The parameters are $\Delta = r^2(r^2 + 2m_0 r)$, $M = \frac{m_0}{2}$,

the cherges are,

$$Q_L^{(a)} = \frac{n^{(a)}}{\sqrt{2}} m_0 \quad 1 \leq a \leq 22, \qquad Q_R^{(a)} = \frac{p^{(a-22)}}{\sqrt{2}} m_0 \quad 23 \leq a \leq 28. \qquad (13)$$

The horizon is at $r = 0$, so $A_H = 0$ and $T_H = \infty$. The gauge fields are given by,

$$\begin{aligned} A_t(a) &= -\frac{n^{(a)}}{\sqrt{2}} \frac{mr^3}{\Delta} \quad & 1 \leq a \leq 22 \\ &= -\frac{p^{(a-22)}}{\sqrt{2}} \frac{mr^3}{\Delta} \quad & 23 \leq a \leq 28 \end{aligned} \qquad (14)$$

The mass saturates the Bogomol'nyi bound $M^2 = \frac{1}{2} \vec{Q}_R^2 = \frac{1}{2} \vec{Q}_L^2$.

We mention in passing that the thermodynamic properties follow naturally from the non-extremal solution. Define $\Phi^{(a)} = A_t^{(a)}|_{r=2m}$ to be the elctrostatic potential at the horizon. Then the following relations hold.

$$T_H d \frac{A_H}{4} = dM - \sum_a \Phi^{(a)} dQ^{(a)}$$

$$T_H \frac{A_H}{2} = M - \sum_a \Phi^{(a)} Q^{(a)} \qquad (15)$$

The explicit form of $\Phi^{(a)}$ is given by,

$$\Phi_L^{(a)} = \frac{n^{(a)}}{\sqrt{2}} \frac{\sinh \alpha}{\cosh \alpha + \cosh \gamma} \qquad 1 \le a \le 22$$

$$\Phi_R^{(a)} = \frac{p^{(a-22)}}{\sqrt{2}} \frac{\sinh \gamma}{\cosh \alpha + \cosh \gamma} \qquad 23 \le a \le 28 \tag{16}$$

For case (I),

$$\Phi_L^{(a)} = 0 \quad 1 \le a \le 22, \qquad \Phi_R^{(a)} = \frac{p^{(a-22)}}{\sqrt{2}} \quad 23 \le a \le 28 \tag{17}$$

and for case (II),

$$\Phi_L^{(a)} = \frac{n^{(a)}}{2\sqrt{2}} \quad 1 \le a \le 22, \qquad \Phi_R^{(a)} = \frac{p^{(a-22)}}{2\sqrt{2}} \quad 23 \le a \le 28 \tag{18}$$

Now we turn to equations of motion,

$$R_{\mu\nu} = \frac{1}{2\lambda_2^2}(\partial_\mu \bar{\lambda}\partial_\nu \lambda + \partial_\nu \bar{\lambda}\partial_\mu \lambda) + \frac{1}{8}Tr(\partial_\mu ML\partial_\nu ML) - g_{\mu\nu}\frac{1}{16}Tr(\partial_\mu ML\partial^\mu ML)$$

$$+ 2\lambda_2 F_{\mu\rho}^{(a)}(LML)_{ab}F_\nu^{(b)\rho} - \frac{1}{2}\lambda_2 g_{\mu\nu}F_{\rho\sigma}^{(a)}(LML)_{ab}F^{(b)\rho\sigma}$$

$$D_\mu(-\lambda_2(ML)_{ab}F^{(b)\mu\nu} + \lambda_1 \tilde{F}^{(a)\mu\nu}) = 0 \tag{19}$$

where $\tilde{F}^{(a)\mu\nu} = \frac{1}{2\sqrt{-g}}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}^{(a)}$ is the usual dual tensor satisfying Bianchi identities.

The first equation in (19) corresponds to the Einstein equation, where R has been eliminated in favour of the matter energy-momentum stress tensor and the second one corresponds to the gauge field equation.

In order to compute the partition function, we need to determine the action on shell. One of the efficient ways to obtain this action is to take the trace of $R_{\mu\nu}$ in (19) to obtain the scalar curvature, R, and use the expression in (1). A straightforward calculation shows that the contributions comes from the well known gravitational boundary term as well as from the gauge field surface term

$$\frac{1}{8\pi} \int_{\Sigma^\infty} \sqrt{-g}d^3x \ n_r[-\lambda_2 A_t^{(a)}(LML)_{ab}F^{(b)tr} + \lambda_1 A_t^{(a)}L_{ab}F^{(b)tr}] \tag{20}$$

7

Now we proceed to compute the partition function for the electrically charged black hole. It is necessary to specify the time integral of the fourth component of the vector potential on the boundary. We follow the prcedure of [8] to carry on this computation for the problem at hand where the effect of 28 gauge bosons are to be taken into account. The gauge potential and the electric field strength have the following form for asymptotically large r

$$\vec{A}_{tL/R} = \frac{\vec{\omega}_{L/R}}{\beta e}(1 - m\frac{\lambda_{L/R}}{r})$$
$$\vec{F}_{trL/R} = \frac{m\lambda_{L/R}\,\vec{\omega}_{L/R}}{\beta e}\frac{1}{r^2} \tag{21}$$

A few comments are in order at this point: the subscript L(R) refer to the gauge fields arising from the compactification of the left(right) moving string coordinates. We recall that 22 gauge fields arise from compactification of left hand sector and other 6 of them come from the right hand sector. Here $\vec{\omega}_{L/R}$ are the generalized version of the parameter introduced in [8]. Here β is the inverse temperature and we have introduced the parameter e to keep track of some power countings; however, note that e will not appear in our expressions for partition function and entropy. The equations (21) are used in (20) to compute the boundary term contributions. The constants $\lambda_{L/R}$ have the following form

$$\lambda_L = \frac{\cosh\gamma}{\cosh\alpha + \cosh\gamma} \qquad \lambda_R = \frac{\cosh\alpha}{\cosh\alpha + \cosh\gamma} \tag{22}$$

The geometry is asymptotically flat and the boundary is taken to be at $r = \infty$. As we have to subtract the flat space contribution according to the prescription of Gibbons and Hawking [10,8], we shall first take the radius to be large and finite and eventually take the limit $r \to \infty$ after the subtraction.

$$S_{boundary} - S_{boundary}^{flat} = \frac{1}{2}\beta(g_{rr})^{-1/2}\partial_r[\Delta^{1/2}\{(g_{rr}^{-1/2} - 1)\}]$$

8

$$= \frac{1}{2}\beta(M - m) \tag{23}$$

where M is the mass of the black hole given by (9). The contribution of (20) together with (23) can be written as

$$Z(\beta, \vec{\omega}) = \exp[-\frac{1}{2}\beta(M - m) - \frac{1}{2e^2}\frac{\lambda_L\vec{\omega}_L^2 + \lambda_R\vec{\omega}_R^2}{4\pi(\cosh\alpha + \cosh\gamma)}] \tag{24}$$

This equation can be interpreted as a grand canonical partition function for electrically charged black holes where $\vec{\omega}$, collectively representing $\vec{\omega}_{L,R}$, play the role of chemical potential. In order to derive the partition function with specific charge configurations, we have to introduce a generalization of the projection technique of [8].

$$Z(\beta, \vec{Q}) = e^{-\beta F(\beta, \vec{Q})}$$
$$= \int_\infty^\infty \Pi_a \frac{d\omega^{(a)}}{2\pi} \exp[-\frac{i}{e}(\vec{\omega}_L \cdot \vec{Q}_L + \vec{\omega}_R \cdot \vec{Q}_R)] \; Z(\beta, \vec{\omega}) \tag{25}$$

where $\vec{Q}$ stands for both $\vec{Q}_L$ and $\vec{Q}_R$. This integral can be evaluated by the saddle point approximation after inserting the expression for $Z(\beta, \vec{\omega})$ given by (24). Thus

$$F(\beta, \vec{Q}) = \frac{1}{2}\beta(M - m) + 2\pi(\cosh\alpha + \cosh\gamma)\left(\frac{\vec{Q}_L^2}{\beta\lambda_L} + \frac{\vec{Q}_R^2}{\beta\lambda_R}\right)$$
$$= \frac{1}{2}(M - m) + \frac{1}{2}(\vec{\Phi}_L \cdot \vec{Q}_L + \vec{\Phi}_R \cdot \vec{Q}_R) \tag{26}$$

where $\Phi_{L/R}$ are the twenty two/ six components of the potentials given in (16). The thermodynamic potential is given by

$$\Omega(\beta, \vec{\Phi}) = M - TS + \vec{\Phi} \cdot \vec{Q}$$
$$= F(\beta, \vec{Q}) - \vec{\Phi} \cdot \vec{Q} \tag{27}$$

Thus the entropy, S is given by

$$S = \beta(M - F)$$
$$= \frac{A_H}{4} + \frac{1}{2}\beta m \tag{28}$$

9

We have used the relation (27) in arriving at (28) . Notice that in both the extremal limits; case (I) and case (II), mentioned earlier, *the entropy vanishes identically.* It is important to note that the quantum corrections to this entropy are vanishing due to the non-renomalization theorems.

Now we proceed to calculate the partition function and entropy of magnetically charged black holes obtained from the electrically charged ones by S-duality transformation. In general axion and dilaton are transformed to new configurations along with the gauge field strengths as given by (4). The electrically charged black hole corresponds to $\lambda_1 = 0$ and ϕ and $A_t^{(a)}$ given by (6) and (7). Our purpose is to implement a special class of duality transformation under which $\lambda_1' = 0$ and $\phi' = -\phi$ and

$$F_{\mu\nu}^{(a)} \rightarrow -\lambda_2(ML)_{ab} F_{\mu\nu}^{(b)}. \tag{29}$$

This is accomplished by a simple choice of parameters $a = d = 0$ and $b = -c = -1$. Note that under this transformation $(\vec{Q}_L, \vec{Q}_R) \rightarrow (-\vec{Q}_L, \vec{Q}_R)$. As the projected partition function for the electric case is quadratic in $\vec{Q}_L$ and $\vec{Q}_R$, the electromagnetic contribution to the partition function will remain unchanged under this transformation. Notice however, the electromagnetic actions corresponding to these two cases differ by a sign. Then we can equate the free energy obtained from this partition function with $M - T\mathcal{S}$ to obtain the expression for the entropy; $\mathcal{S} = \frac{A_H}{4} + \frac{1}{2}\beta m$, which is the same for the electric case.

To summarize: our results derived from the string effective action show that both the extremal black holes have vanishing entropy which is conformitive of the work of Hawking, Horowitz and Ross [11]. In the past it has been proposed that extremal black holes might be identified with elementery particles [12]/ massive states of the string theory [13]. Computation of partition functions and entropy for the extremal case are protected from quantum corrections due to presence of N=4 supersymmetry. S-duality is

beleived to be exact symmetry of string theory [14], which has far reaching consequences. Indeed our work indicates another interesting implication of S-duality.

Acknowledgement

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